Solving Complex Equations

Definition:
A ***Distribution Equation*** is an equations with a two-part number in parentheses which is multiplied.

Strategy: We solve by distributing the multiplier both values inside the parentheses, we combine like-terms, and use opposite-operations. We usually expect one solution.

Distribution Examples:

$$-8.27(7.1x+10.3)=-354.81$$

$$-61.77x-89.61=-354.81$$
$$+89.61 \quad +89.61$$

$$-61.77x=-265.2$$
$$\div -61.77 \quad \div -61.77$$

$$x=4.29$$

$$-31.5(6x-17.85)-10=-924.55$$

$$-189x+562.28-10=-924.55$$

$$-189x+552.28=-924.55$$
$$-552.28 \quad -552.28$$

$$-189x=-1476.83$$
$$\div -189 \quad \div -189$$

$$x=7.81$$

In both cases I began my solving by multiplying the number to the left of the parentheses by both numbers inside the parentheses. After multiplying, I combined like-terms and finished using opposite operations.

Definition:
An ***Exponential Equation*** is an equation which contains at least one base being raised to the power of 'x.' In our class, the x-value will be a whole number.

Strategy: We solve by isolating the base and its variable-exponent. We isolate the base and its exponent using opposite operations. After this isolation we seek out the exponent which makes a true statement from a short-list of powers of the base.

Exponential Examples:

$$4.4\cdot 6^x+19.38=45.78$$
$$-19.38 \quad -19.38$$

$$4.4\cdot 6^x=26.4$$
$$\div 4.4 \quad \div 4.4$$

$$6^x=6$$

$$6^{-1}=\frac{1}{6},\ 6^0=1,\ 6^1=6,\ 6^2=36$$

$$x=1$$

$$-3\cdot 5^x-13.6=-88.6$$
$$+13.6 \quad +13.6$$

$$-3\cdot 5^x=-75$$
$$\div -3 \quad \div -3$$

$$5^x=25$$

$$5^{-1}=\frac{1}{5},\ 5^0=1,\ 5^1=5,\ 5^2=25$$

$$x=2$$

In both examples we began, as our strategy stated, by removing the number being added or subtracted from the base-and-exponent. Having done this, we divided off the multiplier leaving the base-and-exponent lonely. A quick glance at a list of multipliers revealed the matching 'x.'

Definition:
A ***Quadratic Equation*** is an equation which has at least one x^2 in it. We usually expect two solutions (they are the roots).

Strategy: We solve a quadratic equation, usually, by simplifying the equation to a three-part number (trinomial), equaling zero, and then factoring. This yields two factors which then reveal the roots according to the zero-product-property. It is also possible to solve using the quadratic formula. We usually expect two solutions.

Quadratic Examples:

$$3x^2+39x+128=38$$
$$-38 \quad -38$$

$$3x^2+39x+90=0$$
$$\div 3 \quad \div 3$$

$$x^2+13x+30=0$$

$$(x+10)(x+3)=0$$

$$x=-10,\ x=-3$$

$$5x^2-20x-24.25=0.75$$
$$-0.75 \quad -0.75$$

$$5x^2-20x-25=0$$
$$\div 5 \quad \div 5$$

$$x^2-4x-5=0$$

$$(x-5)(x+1)=0$$

$$x=5,\ x=-1$$

In both examples we began by subtracting the useless right-side number away to get the all-important "=0." From this point we targeted the leading coefficient and divided the entire equation by it (even the zero). This left us a nice trinomial to factor. Having seen the factors we assigned the roots, which are the solutions.

Definition:
An ***Absolute-Value Equation*** is an equation with a variable inside absolute-value brackets, measuring a distance to zero. We usually expect two solutions.

Absolute-Value Examples:

$10\left|\frac{7x-2}{3}\right| - 8 = 42$
$+8 \quad +8$

$10\left|\frac{7x-2}{3}\right| = 50$
$\div 10 \quad \div 10$

$\left|\frac{7x-2}{3}\right| = 5$

$\frac{7x-2}{3} = 5 \qquad \frac{7x-2}{3} = -5$
$*3 \quad *3 \qquad *3 \quad *3$

$7x - 2 = 15 \qquad 7x - 2 = -15$
$+2 \quad +2 \qquad +2 \quad +2$

$7x = 17 \qquad 7x = -13$
$\div 7 \quad \div 7 \qquad \div 7 \quad \div 7$

$x = 2.43, \qquad x = -1.86$

$1.25\left|2x - 1.6\right| - 5.7 = 7.22$
$+5.7 \quad +5.7$

$1.25\left|2x - 1.6\right| = 12.92$
$\div 1.25 \quad \div 1.25$

$\left|2x - 1.6\right| = 10.34$

$2x - 1.6 = 10.34 \qquad 2x - 1.6 = -10.34$
$+1.6 \quad +1.6 \qquad +1.6 \quad +1.6$

$2x = 11.94 \qquad 2x = -8.74$
$\div 2 \quad \div 2 \qquad \div 2 \quad \div 2$

$x = 5.97, \qquad x = -4.37$

Strategy: We solve an absolute-value equation first by isolating the bracket–this is done using opposite-operations. Once the bracket is by itself, we break the equation into a positive and a negative version, simultaneously, while dropping the brackets. Both solutions are then solved for using opposite operations. We expect two solutions.

In both examples we began by adding the required number to make a term disappear and then by dividing, leaving the bracket alone. Once done we dismissed the brackets but found a positive and negative version of the equation shown inside the brackets. We found two solutions for each using opposite operations.

Definition:
A ***Radical Equation*** is an equation with a variable inside a radical bracket. We usually expect one solution.

Radical Examples:

$\sqrt{\frac{4x}{8} + 2} + 3 = 15$
$-3 \quad -3$

$\sqrt{\frac{4x}{8} + 2} = 12$

$\left(\sqrt{\frac{4x}{8} + 2}\right)^2 = 12^2$

$\frac{4x}{8} + 2 = 144$
$-2 \quad -2$

$\frac{4x}{8} = 142$
$*8 \quad *8$

$4x = 1136$
$\div 4 \quad \div 4$

$x = 284$

$3.6\sqrt{2.1x + 16.3} + 9.7 = 24.3$
$-9.7 \quad -9.7$

$3.6\sqrt{2.1x + 16.3} = 14.6$
$\div 3.6 \quad \div 3.6$

$\sqrt{2.1x + 16.3} = 4.06$

$\left(\sqrt{2.1x + 16.3}\right)^2 = 4.06^2$

$2.1x + 16.3 = 16.48$
$-16.3 \quad -16.3$

$2.1x = 0.18$
$\div 2.1 \quad \div 2.1$

$x = 0.09$

Strategy: We solve a radical equation by isolating the radical bracket using opposite operations. Once isolated, the bracket is squared away (the opposite side too is squared) and the remaining equation can be simplified. We expect one solution.

As our strategy indicated, we began by subtracting away confusing values outside the brackets. Division was necessary for the right. Both equations were then squared, removing the bracket and solved for ‘x’ using very basic equation-solving-skills.

Definition:
A ***Proportion Equation*** is an equation in which two fractions are set equal to each other and in which there is at least one variable. We usually expect one solution.

Strategy: We solve by cross-multiplying the top-left by the bottom right (by two parts if necessary) and then by cross-multiplying the top-right by the bottom-left (again by two parts if necessary). We isolate the 'x' by using opposite operations. We usually expect one solution.

Proportion Examples:

$$\frac{4x+5.7}{x-16}=\frac{47.5}{8}$$

$$8(4x+5.7)=47.5(x-16)$$

$$32x+45.6=47.5x-760$$

-32x -32x

+760 +760

$$805.6=15.5x$$

÷ 15.5 ÷ 15.5

$$x=51.97$$

$$\frac{2x-13.4}{3x+6.85}=\frac{13.45}{39.56}$$

$$39.56(2x-13.4)=13.45(3x+6.85)$$

$$79.12x-530.1=40.35x+92.13$$

-40.35x -40.35x

+530.1 +530.1

$$38.77x=622.23$$

÷ 38.77 ÷ 38.77

$$x=16.05$$

In both examples we began by cross-multiplying the one-part number (monomial) by both parts of the two-part number (binomial). The cross multiply resulted in a very ordinary equation which we solved using opposite operations.

A ***Complex-notation Equation*** is an equation in which a number's [decimal] value is concealed as a fraction, an absolute-value, an exponent, or a radical.

Strategy: Our strategy is really simple in these examples. We convert each piece of non-decimal number into easily handled decimals (I am rounding to hundredths). After this we combine like-terms and use opposite operations.

Complex-notation Examples:

$$19.56x+\frac{35}{15}+8.4^2=\sqrt{414}+|-31.2|$$

$$19.56x+2.33+70.56=20.35+31.2$$

$$19.56x+72.89=51.55$$

-72.89 -72.89

$$19.56x=-21.34$$

÷ 19.56 ÷ 19.56

$$x=-1.09$$

$$6.1^4+2x+\sqrt{137.6}=\frac{543}{27}+9.4x-|-58|$$

$$1384+2x+11.73=20.11+9.4x-58$$

$$2x+1395.73=9.4x-37.89$$

-2x -2x

+37.89 +37.89

$$1433.62=7.4x$$

÷ 7.4 ÷ 7.4

$$x=193.73$$

In both examples we saw that we were left with a number of x's and four sets of decimals. We had to spend a step combining, but then it was a matter of simple adding and subtracting, completed with a division step.