Reading Exponential Functions from a Graph

Definitions:
An **Exponential Function** is a relationship of numbers in which a positive base is raised to powers other than one.
There are four types of exponential curves.

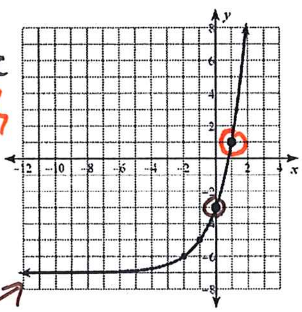
Formulas:
$y = ab^x + c$ or
$f(x) = ab^x + c$

a is an initial value or coefficient.
b is the base being raised.
c is the horizontal asymptote.

Example #1:

$y=a\cdot b^x+c$
$-3=a\cdot b^0-7$
$+7 \quad +7$
$4=a\cdot 1$
$a=4$
$b=2$
$c=-7$

$y=a\cdot b^x+c$
$1=4\cdot b^1-7$
$+7 \quad +7$
$8=4b$
$\frac{8}{4}=\frac{4b}{4}$
$b=2$
$\boxed{y=4\cdot 2^x-7}$



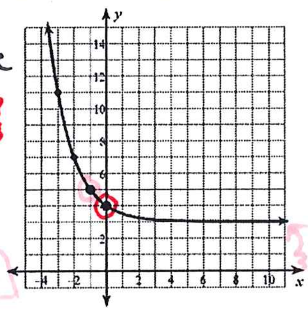
$c=-7$

Working to find the exponential formula off the graph is very similar to finding the formula off a table. The graph provides several whole-number points and like the table we want to read our asymptote and apply it to the c-value in the equation, and we want the best adjacent points, if possible, using the x=0 and x=1.
Ex #1 presents an Exponential Growth Curve.

Example #2:

$y=a\cdot b^x+c$
$5=a\cdot b^{-1}+3$
$-3 \quad -3$
$2=a\cdot b^{-1}$
$2=1\cdot b^{-1}$
$a=1$
$b=.5$
$c=3$

$y=a\cdot b^x+c$
$4=a\cdot b^0+3$
$-3 \quad -3$
$1=a\cdot 1$
$b=\frac{1}{2}$
$\boxed{y=1\cdot\left(\frac{1}{2}\right)^x+3}$



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Example #2 is a little more difficult because we do not get to use an x=1 value. There is no whole number to draw from and we cannot assume any particular fraction. As such we will use the x=0 first (0,4), and then the adjacent x=-1 second (-1,5). We will finish using a reciprocal.

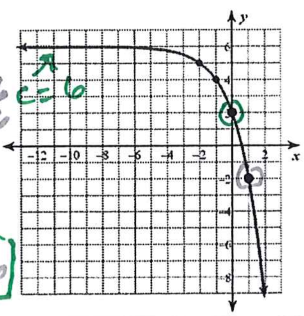
Ex #2 presents an Exponential Decay Curve.

Example #3:

$y=a\cdot b^x+c$
$2=a\cdot b^0+6$
$-6 \quad -6$
$-4=a\cdot 1$
$a=-4$
$b=2$
$c=6$

$y=a\cdot b^x+c$
$-2=-4\cdot b^1+6$
$-6 \quad -6$
$\frac{-8}{-4}=\frac{-4\cdot b}{-4}$
$b=2$
$\boxed{y=-4\cdot 2^x+6}$

$c=6$



Example #3 again opens the door to using the x=0 and the x=1. We begin using our horizontal asymptote and the whole-number points to find the 'a' and 'b' values.

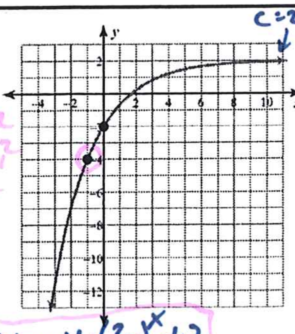
Ex #3 presents an Inverse Growth Curve.

Example #4:

$y=a\cdot b^x+c$
$-2=a\cdot b^0+2$
$-2 \quad -2$
$-4=a\cdot 1$
$a=-4$
$b=\frac{2}{3}$
$c=2$

$y=a\cdot b^x+c$
$-4=-4\cdot b^{-1}+2$
-2
$\frac{-6}{-4}=\frac{-4\cdot b^{-1}}{-4}$
$\frac{3}{2}=b^{-1}$
$b^1=\frac{2}{3}$

$c=2$



$\boxed{y=-4\cdot\left(\frac{2}{3}\right)^x+2}$

Example #4 denies us a x=1 but we can use the adjacent points x=0 and x=-1. We do have to finish up with a reciprocal.

Ex #4 presents an Inverse Decay Curve.

Example #5:

$y=a\cdot b^x+c \qquad y=a\cdot b^x+c$

$8=a\cdot b^1+3 \qquad 13=a\cdot b^2+3$

$-3 \quad -3 \qquad -3 \quad -3$

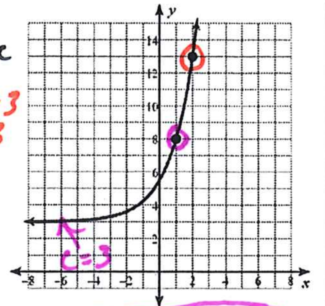
$5=a\cdot b^1 \qquad 10=a\cdot b^2$

$$\frac{{}^{2}\cancel{10}=\cancel{a}\cdot\cancel{b}\cdot b}{{}^{1}\cancel{5}=\cancel{a}\cdot\cancel{b}}$$

$a=\frac{5}{2}$ $\qquad 5=a\cdot 2$

$b=2$ $\qquad \div 2 \quad \div 2$

$c=3$ $\qquad a=\frac{5}{2}$



$\boxed{y=\frac{5}{2}\cdot(2)^x+3}$

Example #5 is tougher than the previous four because we do not have the coveted x=0, which makes life easy. We still have the ability to pick consecutive points, x=1 and x=2, and their y-values.

*Once we set up the two equations we solved to 5=a*b and 10=a*b*b (which was the same as b^2).*

$$\frac{10=a*b*b}{5=a*b}$$ *The fraction divides down to b=2.*

Ex #5 presents an Exponential Growth Curve.

Example #6:

$y=a\cdot b^x+c \qquad y=a\cdot b^x+c$

$-3=a\cdot b^{-1}-4 \qquad -2=a\cdot b^{-2}-4$

$+4 \quad +4 \qquad +4 \quad +4$

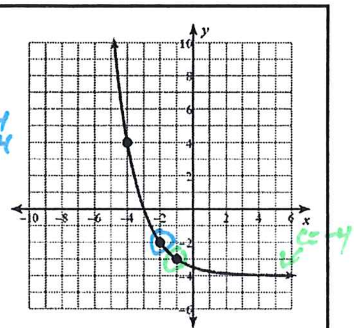
$1=a\cdot b^{-1} \qquad 2=a\cdot b^{-2}$

$$\frac{1=\cancel{a}\cdot\cancel{b^{-1}}}{2=\cancel{a}\cdot\cancel{b^{-1}}\cdot b^{-1}}$$

$a=\frac{1}{2}$ $\qquad 2=b^{-1}$

$b=\frac{1}{2}$ $\qquad b^1=\frac{1}{2}$

$c=-4$



$\boxed{y=\frac{1}{2}\cdot(\frac{1}{2})^x-4}$

Example #6 follows much the same process but without the benefit of positive x-values. I was required to use '-1' for x in one equation and '-2' in the other.

That did not effect my ability to divide the two equations though, because b^{-1} does divide by b^{-1}.

$$\frac{2=a*b^{-1}*b^{-1}}{1=a*b^{-1}}$$ *The fraction divides down to $b^{-1}=2$, Meaning that we require the reciprocal, ½ for our b-value.*

Ex #6 presents an Exponential Decay Curve.

Example #7:

$y=a\cdot b^x+c \qquad y=a\cdot b^x+c$

$3=a\cdot b^1+8 \qquad -2=a\cdot b^2+8$

$-8 \quad -8 \qquad -8 \quad -8$

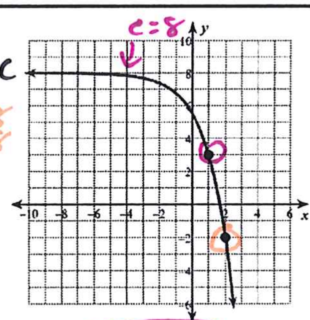
$-5=a\cdot b \qquad -10=a\cdot b^2$

$$\frac{{}^{2}\cancel{-10}=\cancel{a}\cdot\cancel{b}\cdot b}{{}^{1}\cancel{-5}=\cancel{a}\cdot\cancel{b}}$$

$a=\frac{-5}{2}$ $\qquad b=2$

$b=2$ $\qquad -5=a\cdot 2$

$c=8$ $\qquad \div 2 \quad \div 2$



$\boxed{y=\frac{-5}{2}\cdot 2^x+8}$

Example #7 is a touch easier than #6 because the x-values are again positive.

After solving for 'b,' we chose the shorter equation to place the value of 'b' into, thus finding our 'a.'

Ex #7 presents an Inverse Growth Curve.

Example #8:

$y=a\cdot b^x+c \qquad y=a\cdot b^x+c$

$-6=a\cdot b^{-1}-4 \qquad -10=a\cdot b^{-2}-4$

$+4 \quad +4 \qquad +4 \quad +4$

$-2=a\cdot b^{-1} \qquad -6=a\cdot b^{-2}$

$$\frac{{}^{3}\cancel{-6}=\cancel{a}\cdot\cancel{b^{-1}}\cdot b^{-1}}{{}^{1}\cancel{-2}=\cancel{a}\cdot\cancel{b^{-1}}}$$

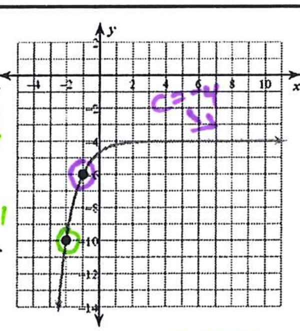
$3=b^{-1}$

$b=\frac{1}{3}$

$a=\frac{-2}{3}$ $\qquad -2=a\cdot 3$

$b=\frac{1}{3}$ $\qquad \div 3 \quad \div 3$

$c=-4$



$\boxed{y=\frac{-2}{3}\cdot(\frac{1}{3})^x-4}$

Example #8 again presented us with the need to use negative exponents because the only whole-number x's were on the left side.

As in example six, we divided the longer equation by the shorter and then used the same shorter equation to find our 'a.'

Once again, we isolated our b-value using a reciprocal.

Ex #8 presents an Inverse Decay Curve.