Finding an Exponential Function from a Data Table

Definitions:
An **Exponential Function** is a relationship of numbers in which a positive base is raised to powers other than one.

Formulas:
$y = ab^x + c$ or
$f(x) = ab^x + c$

a is an initial value or coefficient.
b is the base being raised.
c is the horizontal asymptote.

Example #1:

x	-3	-2	-1	0	1	2	3	4	5
y	6.07	6.22	6.67	8	12	24	60	168	492

asymptote

$y = a \cdot b^x + c$
8 0 6
$8 = a \cdot b^0 + 6$
$-6 \qquad -6$
$2 = a \cdot 1$
$a = 2$

12 2 1 6
$y = a \cdot b^x + c$
$12 = 2 \cdot b^1 + 6$
$-6 \qquad -6$
$\frac{6}{2} = \frac{2 \cdot b}{2}$
$3 = b$

$\boxed{y = 2 \cdot 3^x + 6}$

My first job, when looking for the formula in the table, was to determine the horizontal asymptote. Since I saw that the y-values were moving closer and closer to six, I am calling my asymptote '6.' There are no decimal asymptotes in this class.

Example #2:

x	-3	-2	-1	0	1	2	3	4	5
y	9.02	9.12	9.60	12	24	84	384	1884	9384

asymptote

$y = a \cdot b^x + c$
12 0 9
$12 = a \cdot b^0 + 9$
$-9 \qquad -9$
$3 = a \cdot 1$
$a = 3$

$y = a \cdot b^x + c$
24 3 1 9
$24 = 3 \cdot b^1 + 9$
$-9 \qquad -9$
$\frac{15}{3} = \frac{3b}{3}$
$b = 5$

$\boxed{y = 3 \cdot 5^x + 9}$

In example two, I identified my asymptote at '9' because of the values I saw on the table and then I picked out the best two points to use in two copies of the formula. The first and best value is whatever comes with the x-value of zero. This is useful because the b^x will always equal '1' when x is zero. We know that all numbers to the zero are one.

Example #3:

x	-3	-2	-1	0	1	2	3	4	5
y	7.38	7.75	8.5	10	13	19	31	55	103

asymptote

$y = a \cdot b^x + c$
10 0 7
$10 = a \cdot b^0 + 7$
$-7 \qquad -7$
$3 = a \cdot 1$
$a = 3$

$y = a \cdot b^x + c$
13 3 1 7
$13 = 3 \cdot b^1 + 7$
$-7 \qquad -7$
$\frac{6}{3} = \frac{3b}{3}$
$b = 2$

$\boxed{y = 3 \cdot 2^x + 7}$

The next-best point to choose after the x=0 is, if I can get it, the x=1 value. The second point gives us the clue we need to unlock not just the a-value but also the b-value. Anytime I can, I want to use adjacent points (values from x's right next to each other). Usually the x=0 and x=1 are the two best to use.

Example #4:

x	-4	-3	-2	-1	0	1	2	3	4
y	158	50	14	2	-2	-3.33	-3.78	-3.93	-3.98

asymptote
$c = -4$

$y = a \cdot b^x + c$
-2 0 -4
$-2 = a \cdot b^0 - 4$
$+4 \qquad +4$
$2 = a \cdot 1$
$a = 2$

$y = a \cdot b^x + c$
2 2 -1 -4
$2 = 2 \cdot b^{-1} - 4$
$+4 \qquad +4$
$\frac{6}{2} = \frac{2 \cdot b^{-1}}{2}$ $\quad b^{-1} = 3 \quad b^1 = \frac{1}{3}$

$\boxed{y = 2 \cdot \left(\frac{1}{3}\right)^x - 4}$

My table is very different this time because my asymptote is on the right-side and is negative, getting closer and closer to '-4.'
In the event that I do not have good whole-number x=0 and x=1 values, I might use an x=-1 instead. I can will have to do this using a reciprocal at the end of my calculations to turn the b^{-1} into a b^1.